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INFLUENCE OF A STELLAR WIND ON THE EVOLUTION OF A STAR OF 30 $M_\odot$

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ABSTRACT

A coarse grid of theoretical evolutionary tracks has been computed for a star of 30 $M_\odot$, in an attempt to delineate the role of mass loss in the star's evolution during core helium burning. For all of the tracks, Cox-Stewart opacities have been adopted, and the free parameters have included the rate of mass loss, criterion for convection, and initial chemical composition. With the use of the Schwarzschild criterion, the star suffers little mass loss during core helium burning and remains almost to the end, a blue supergiant, well separated from main-sequence stars on the H-R diagram. With the use of the Ledoux criterion, the same consequences are obtained only in the case of a relatively low initial hydrogen or initial metals abundance. Otherwise, the star evolves, first, into a red supergiant, whereupon rapid mass loss must be assumed to take place, if the observed paucity of very bright red supergiants is to be accounted for. The stellar remnant then consists of little more than the original helium core, and may appear, for a time, bluer than equally luminous main-sequence stars, provided that the initial hydrogen and metals abundances are normal. Thus, a wide variety of possible evolutionary tracks can be obtained for an initial mass of 30 $M_\odot$ with reasonable choices of the free parameters.

Subject headings: stars: evolution — stars: interiors — stars: mass loss — stars: winds

I. INTRODUCTION

The stellar winds observed in the brightest main-sequence stars are so strong that existing uncertainties in the measured rates of mass loss have upset theoretical efforts to determine how these stars will evolve. For the time being, therefore, it may be advantageous to study the evolution of stars not quite so massive, which lie slightly farther down the main sequence. If the amount of mass lost during the main-sequence phase of evolution can be assumed to be small for these stars, their future evolution can be studied with much greater confidence. With this purpose in mind, we have undertaken a study of stellar evolution at 30 $M_\odot$, in the expectation that the effects of mass loss will be significant and interesting in the post-main-sequence stages of evolution, but not in the main-sequence stages.

For this work, we have adopted Cox-Stewart opacities, although the use of the Carson opacities would give quite different results. Earlier papers of ours have discussed the physical interpretation of the various types of evolutionary tracks that are obtainable for stars of 30 $M_\odot$ (e.g., Stothers and Chin 1975, 1976, 1978, 1979); so we shall simply present here the new numerical results, which have been derived for a wide range of free parameters, including the rate of mass loss, criterion for convection, and initial chemical composition. In §§ II and III, the main assumptions of the present paper are set out. In § IV, a grid of evolutionary tracks based on the Schwarzschild criterion for convection is presented, while in § V a grid based on the Ledoux criterion is examined. A short summary of our main results concludes that paper.

II. ANALYTIC REPRESENTATION OF THE RATE OF MASS LOSS

Bearing in mind the large uncertainties of the observed rates of mass loss, we shall find it useful to approximate the observed rates by a simple formula.

For *early-type* stars, such a formula ought to satisfy the following requirements: (1) the rates along the zero-age main sequence should be accurately reproduced; (2) the rates just off the zero-age main sequence should increase sharply with radius, at constant luminosity; and (3) the rates at very large radii should depend only on luminosity. The observational material bearing on these requirements is provided by Hutchings (1976), Barlow and Cohen (1977), Conti and Garmany (1980), and their references.

A formula satisfying all the requirements is

$$-\frac{dM}{dt} = \frac{aL^\alpha R^\beta M^\gamma}{1 + bL^\delta R^\beta M^\gamma}, \qquad (1)$$

where $\alpha = 1$, $\beta = 4$, $\gamma = -2$, $\delta = 0$, $a = 5 \times 10^{-14}$, and $b = 5 \times 10^{-3}$, provided that the physical variables are expressed in solar units. This formula represents the observations to within a factor of 2, over the range $4.6 \leq \log (L/L_\odot) \leq 5.7$ and $3.9 \leq \log T_e \leq 4.7$. Notice that, along the zero-age main sequence (where the surface gravity, GM/R^2, is approximately constant) and among the blue supergiants with very large radii, the rate of mass loss obeys the Fesenkov (1949) law, $-dM/dt \propto L$. For significantly brighter luminosities (beyond the scope of this paper), formulae have been provided by Abbott *et al.* (1980) and by Chiosi (1980).

If, as Abbott *et al.* (1980) have suggested for extremely luminous O stars, the true rates of mass loss are independent of stellar radius even for *ordinary* O stars, equation (1) will be found to predict rates that are too small along the zero-age main sequence. In view of this uncertainty, it may actually be best to fall back on the simple Fesenkov law or else on the somewhat less restrictive McCrea (1962) formula,

$$-\frac{dM}{dt} = \frac{kLR}{M}, \tag{2}$$

with the value of k adjusted so that the formula gives an upper limit to the true rates. Such a value is $k = 1 \times 10^{-11}$.

In the case of the *late-type* supergiants, we are confronted with even greater uncertainties. Kudritzski and Reimers (1978) have represented their observations by the McCrea formula with $k = 5 \times 10^{-13}$. Their rates agree to within a factor of 5 with rates derived by Weymann (1962), Gehrz and Woolf (1971), Sanner (1976), and Hagen (1978). However, the rates determined by Bernat (1977) rise more steeply with luminosity than does the McCrea formula; thus from his work k is an increasing function of luminosity and reaches a value of about 1×10^{-11} at the brightest observed luminosities. Since even brighter luminosities are possible for stars of initially 30 $M_\odot$, the k value that we need in the present work could be indeterminately larger.

For the purposes of later discussion, we shall define three general cases of mass loss, in conformity with our earlier work: in case A, no mass loss occurs at all; in case B, mass loss occurs at all stages of evolution with a rate given by equation (2); and in case C, mass loss occurs only among relatively cool supergiants with $\log T_e < 3.85$; the rate is given by equation (2).

In cases B and C, the adopted values of k will be specified below.

III. OTHER ASSUMPTIONS

Essentially, we have followed the procedures and assumptions adopted in our previous work. It is perhaps worth repeating here that we have assumed, as is now standard, that the same criterion for convection applies to the state of convective neutrality (i.e., to semiconvection) as to the outbreak of convection. Both the Schwarzschild criterion and the Ledoux criterion have been employed here.

Notation in the present paper is as follows: X_e, initial hydrogen abundance; Z_e, initial metals abundance; α_P, ratio of convective mixing length to pressure scale height; $\log T_e$ (ZAMS), logarithm of the hottest effective temperature occurring during the phase of core hydrogen burning; $\log T_e$ (TAMS), logarithm of the coolest effective temperature occurring during the phase of core hydrogen burning; $\log T_e$ (tip), logarithm of the hottest effective temperature that is achieved during the *slow* stages of core helium burning; $\log T_e(b/y)$, logarithm of the transitional effective temperature that divides the slow *blue* stages of core helium burning from the fast *yellow* stages; τ_H, lifetime of core hydrogen burning; τ_{He}, lifetime of core helium

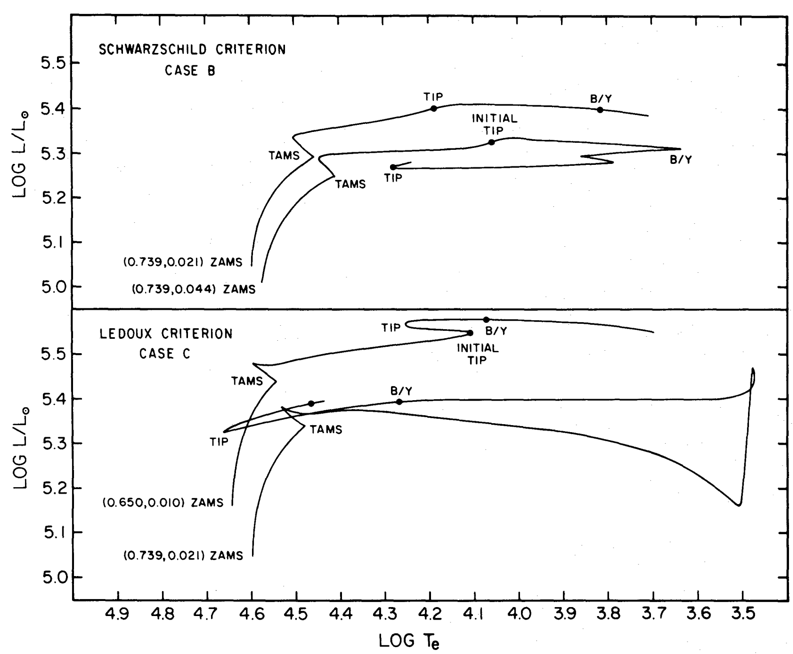

FIG. 1.—Examples of the four different types of evolutionary tracks in the H-R diagram, for a star of initially 30 $M_\odot$. Each track is labeled with the adopted value of (X_e, Z_e). In all cases shown, the mass-loss parameter k is chosen to be 1×10^{-11}. Heavy dots mark the beginning and end of the slow *blue* stages of core helium burning.

burning, as measured from the instant of central hydrogen exhaustion; τ_b/τ_{He} and τ_r/τ_{He}, fractions of the helium-burning lifetime that are spent in the blue stages and red stages, respectively.

The yellow stages turn out, in all cases, to be so rapid that they may be simply included as part of the red stages. But since the transition between blue and yellow stages is never very sharp, the estimated values of $\log T_e(b/y)$ cannot be accorded much precision. Illustrations of the four different types of evolutionary tracks that have been encountered in the present work are shown in Figure 1. The full survey is based on the following input parameters (apart from the rate of mass loss): $X_e = 0.602$–0.739, $Z_e = 0.010$–0.044, and $\alpha_P = 1$.

IV. EVOLUTIONARY TRACKS BASED ON THE SCHWARZSCHILD CRITERION FOR CONVECTION

Since it is already known that evolutionary tracks based on the Schwarzschild criterion for convection remain blue throughout most of the phase of core helium burning even if the rate of mass loss is relatively high (Stothers and Chin 1979; Falk and Mitalas 1979), we decided first to compute a track with initial composition parameters $(X_e, Z_e) = (0.739, 0.021)$ for the case where the rate of loss is given by equation (1). The results for this case closely resemble those previously obtained for the case where mass loss is neglected. More precisely, the star loses about 3% of its initial mass during the main-sequence phase, another 5% disappearing during the phase of core helium burning.

Although this would suggest that case A is probably a good approximation, the true rate of mass loss is uncertain enough to justify using equation (2) with $k = 1 \times 10^{-11}$ as an upper limit. Accordingly, a grid of evolutionary tracks has been computed both for case A and for case B, the latter case having $k = 1 \times 10^{-11}$. Results are shown in Table 1 and Figure 2, which include a small number of previously published tracks (Stothers and Chin 1976, 1979). The main-sequence bands in the figure encompass all the derived results for the phase of core hydrogen burning that are based on the various assumed initial chemical compositions.

It is quickly apparent that the band of helium-burning supergiants is always well separated from the main-sequence band. Therefore, the assumption of a wide range of initial chemical compositions and the adoption of the largest plausible mass-loss rates fail to account for the observed absence of a division of the early-type stars in this part of the H-R diagram.

Masses and surface hydrogen abundances at the final stages of our evolutionary tracks for case B are displayed in Figure 3. Since these results represent our most extreme case, we, like others (de Loore, De Grève, and Lamers 1977; Czerny 1979; Massevitch *et al.* 1979), find that, for an initial mass of 30 $M_\odot$, mass loss during the main-sequence and blue-supergiant phases is likely to be insignificant. Therefore, we feel some doubt about the heavy mass loss advocated by Chiosi, Nasi, and Sreenivasan (1978), Chiosi, Nasi, and Bertelli (1979), and Dearborn and Blake (1979) for stars of this type.

TABLE 1

EVOLUTIONARY SEQUENCES OF MODELS FOR STARS OF INITIALLY 30 $M_\odot$ BASED ON THE SCHWARZSCHILD CRITERION FOR CONVECTION

Case	X_e	Z_e	k (10^{-11} $M_\odot$ yr^{-1})	$\log T_e$ (ZAMS)	$\log T_e$ (TAMS)	$\log T_e$ (tip)	$\log T_e$ (b/y)	τ_H (10^6 yr)	τ_{He}/τ_H	τ_b/τ_{He}	Final $M/M_\odot$
A	0.739	0.010	0	4.62	4.51	4.29	~4.13	5.998	0.084	0.958	30.0
	0.739	0.021	0	4.60	4.49	4.24	~4.07	6.101	0.086	0.968	30.0
	0.739	0.044	0	4.57	4.45	4.09	~3.75	6.403	0.084	0.902	30.0
	0.650	0.010	0	4.64	4.55	4.29	~4.10	4.659	0.100	0.951	30.0
	0.650	0.021	0	4.63	4.52	4.23	~3.94	4.705	0.104	0.953	30.0
	0.650	0.044	0	4.60	4.48	4.15	~3.91	4.821	0.100	0.944	30.0
	0.602	0.010	0	4.66	4.57	4.29	~4.01	4.027	0.115	0.952	30.0
	0.602	0.021	0	4.64	4.54	4.26	~4.09	4.110	0.116	0.920	30.0
	0.602	0.044	0	4.61	4.51	4.14	~3.85	4.182[a]	0.111	0.993	30.0
B............	0.739	0.010	1	4.62	4.49	4.28	~4.03	5.854	0.090	0.956	22.8
	0.739	0.021	1	4.60	4.46	4.19	~3.82	5.945	0.089	0.975	17.5
	0.739	0.044	1	4.57	4.41	4.27[b]	3.63	6.220	0.086	1.000	10.0
	0.650	0.010	1	4.64	4.54	4.29	~4.00	4.529	0.109	0.953	21.0
	0.650	0.021	1	4.63	4.50	4.19	~3.96	4.630	0.106	0.988	17.8
	0.650	0.044	1	4.60	4.46	4.21[c]	3.78	4.796	0.101	1.000	11.5
	0.602	0.010	1	4.66	4.56	4.30	~4.05	3.917	0.120	0.968	20.7
	0.602	0.021	1	4.64	4.53	4.21	3.99	3.991	0.123	1.000	18.0
	0.602	0.044	1	4.61	4.48	4.16[d]	3.85	4.134	0.115	1.000	12.1

[a] Corrected value (cf. Stothers and Chin 1976).

[b] Initial $\log T_e$(tip) = 4.06.

[c] Initial $\log T_e$(tip) = 4.08.

[d] Initial $\log T_e$(tip) = 4.10.

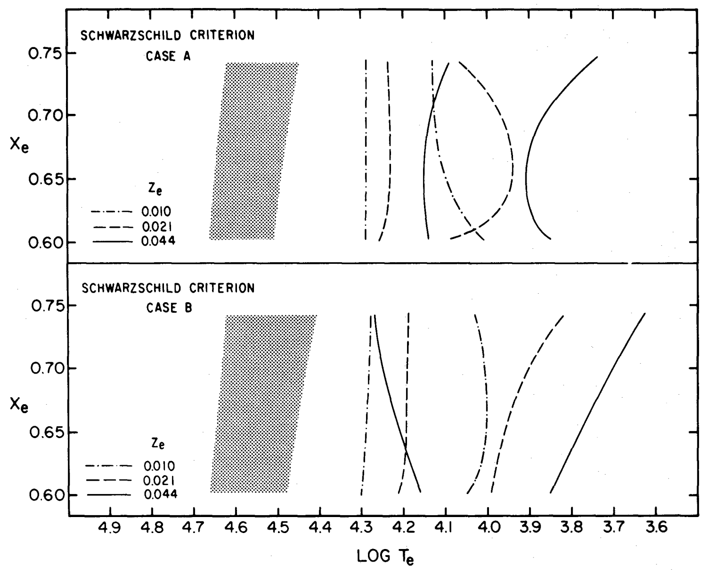

FIG. 2.—Hottest and coolest effective temperatures of the blue supergiants evolving in the slow stages of core helium burning, as a function of initial chemical composition. The Schwarzschild criterion for convection has been adopted. Dotted regions define the main-sequence band of stars burning core hydrogen. In the lower panel, k has been taken to be 1×10^{-11}.

V. EVOLUTIONARY TRACKS BASED ON THE LEDOUX CRITERION FOR CONVECTION

A preliminary track based on the Ledoux criterion for convection was first computed for initial composition parameters $(X_e, Z_e) = (0.739, 0.021)$ in the prior knowledge that the track would enter the red region of the H-R diagram during the onset of core helium burning. Mass was removed from the star (in the red region) at a rate given by equation (2) with $k = 5 \times 10^{-13}$. This example of case C evolution yielded results very close to the results for case A evolution (Stothers and Chin 1979). The star remained red throughout the phase of core helium burning and lost only 13% of its initial mass.

However, the adopted k value, 5×10^{-13}, is merely an average observed value that was derived from measurements of stars with lower luminosities than those being considered here. The true rate of mass loss from a star of initially 30 $M_\odot$ may be much greater. In fact, very high rates of mass loss would be needed in order to bring the star quickly out of the red-supergiant state, in which few, if any, supergiants of high luminosity are actually observed. Although without any mass loss the star may still evolve out of the red-supergiant state for certain values of the initial parameters, the transition always seems to occur after a considerable lapse of time (Ziolkowski 1972; Stothers and Chin 1975, 1979). Therefore, case A evolution is of no practical interest here, and will be ignored.

Accordingly, evolutionary tracks have been computed for case C evolution with $k = 1 \times 10^{-11}$, 3×10^{-11}, and 10×10^{-11}. In order to assess the possible influence of mass loss in the blue region of the H-R diagram, a few additional tracks have been computed for case B evolution with $k = 1 \times 10^{-11}$.

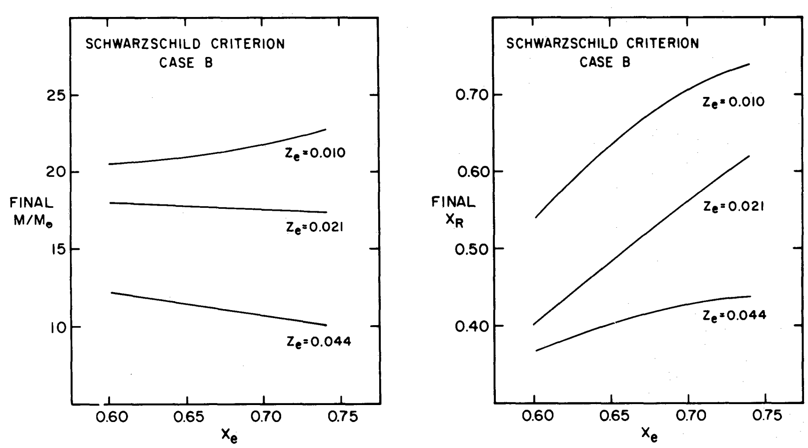

FIG. 3.—Final stellar mass and final surface hydrogen abundance at the end of core helium burning, as a function of initial chemical composition. The Schwarzschild criterion for convection has been adopted, and k has been taken to be 1×10^{-11}.

TABLE 2

EVOLUTIONARY SEQUENCES OF MODELS FOR STARS OF INITIALLY 30 $M_\odot$ BASED ON THE LEDOUX CRITERION FOR CONVECTION

Case	X_e	Z_e	k (10^{-11} $M_\odot$ yr^{-1})	log T_e (ZAMS)	log T_e (TAMS)	log T_e (tip)	log T_e (b/y)	τ_H (10^6 yr)	τ_{He}/τ_H	τ_b/τ_{He}	Final $M/M_\odot$
C.	0.739	0.010	1	4.62	4.51	4.46	~4.02	5.679	0.083	0.789	12.8
	0.739	0.021	1	4.60	4.49	4.67	~4.27	5.763	0.083	0.831	11.3
	0.739	0.021	3	4.60	4.49	4.83	~4.21	5.763	0.082	0.914	11.2
	0.739	0.021	10	4.60	4.49	4.84	~3.86	5.763	0.082	0.948	11.3
	0.739	0.044	1	4.57	4.45	4.76	~4.41	5.988	0.080	0.807	10.4
	0.739	0.044	3	4.57	4.45	4.88	~4.24	5.988	0.080	0.915	10.3
	0.739	0.044	10	4.57	4.45	4.89	~4.36	5.988	0.081	0.950	10.2
	0.650	0.010	any	4.64	4.55	4.26[a]	~4.08	4.432	0.101	0.953	30.0
	0.650	0.021	1	4.63	4.52	4.53	~4.23	4.500	0.098	0.828	13.4
	0.650	0.044	1	4.60	4.49	4.73	~4.13	4.748	0.095	0.826	11.7
	0.602	0.010	any	4.66	4.57	4.28[b]	~4.12	3.833	0.114	0.943	30.0
	0.602	0.021	1	4.64	4.54	4.33[c]	~4.07	3.869	0.109	0.836	15.6
	0.602	0.021	3	4.64	4.54	4.48[c]	~4.14	3.869	0.110	0.900	15.0
	0.602	0.021	10	4.64	4.54	4.55[c]	~4.19	3.869	0.110	0.925	14.7
	0.602	0.044	1	4.61	4.51	4.75	~4.30	4.010	0.106	0.843	12.6
B.	0.739	0.021	1	4.60	4.46	4.90	~4.29	5.959	0.083	0.848	11.0
	0.650	0.010	1	4.64	4.53	4.83	~4.30	4.499	0.101	0.846	13.1
	0.602	0.010	1	4.66	4.56	4.85	~4.11	3.909	0.111	0.861	13.8
	0.602	0.021	1	4.64	4.53	4.91	~4.14	3.982	0.110	0.875	13.1

[a] This sequence remained blue; initial log T_e(tip) = 4.11.

[b] This sequence remained blue; initial log T_e(tip) = 4.16.

[c] This sequence became red during the onset of core helium burning, in contrast to an earlier calculation (cf. Stothers and Chin 1975).

Table 2 and Figure 4 contain the main results (one track is taken from Stothers and Chin 1979).

The most important result obtained from these tracks is the enormous width of the band in the H-R diagram occupied by the hot descendants of the supergiants. This band extends so far toward the blue that, if X_e, Z_e, or k is large enough, it overlaps the main-sequence band. Observations, as we have mentioned, seem to favor such a continuous distribution of stars. But these hot blue remnants may possibly become *too* blue if mass loss on the blue side of the H-R diagram is important and strips away the residual hydrogen envelope (see the case B results in Table 2).

Masses and surface hydrogen abundances of the blue remnants in case C are shown in Figure 5. Since these masses are not much larger than the helium core masses at the end of the main-sequence phase of evolution, the final trends with X_e and Z_e reflect

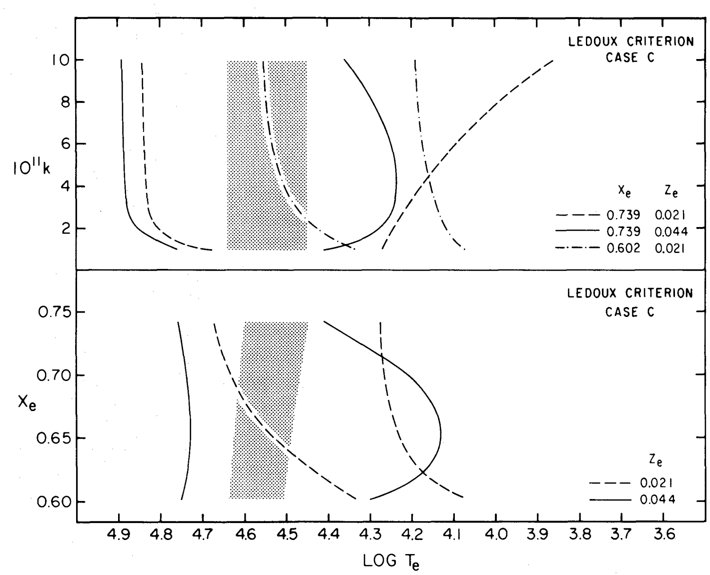

FIG. 4.—Hottest and coolest effective temperatures of the blue supergiants evolving in the slow stages of core helium burning, as a function of initial chemical composition and mass-loss parameter k. The Ledoux criterion for convection has been adopted. Dotted regions define the main-sequence band of stars burning core hydrogen. In the lower panel, k has been taken to be 1×10^{-11}.

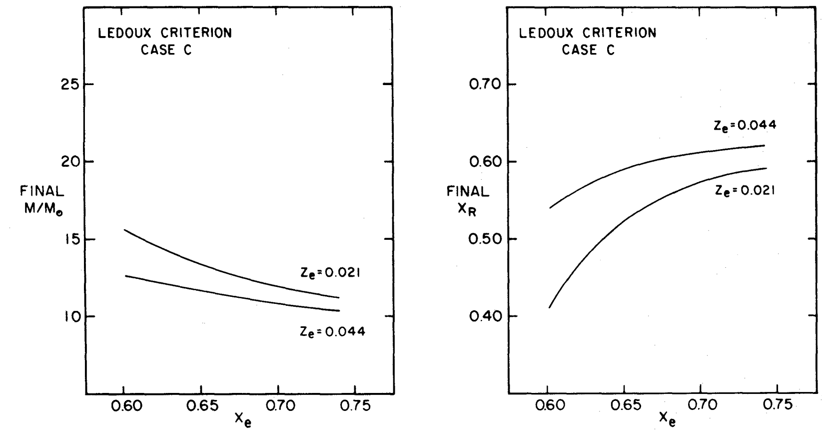

FIG. 5.—Final stellar mass and final surface hydrogen abundance at the end of core helium burning, as a function of initial chemical composition. The Ledoux criterion for convection has been adopted, and k has been taken to be 1×10^{-11}.

primarily main-sequence evolution. Somewhat paradoxically, the remnants that become bluest have the largest surface hydrogen abundances. The reason for this anomaly is that the hydrogen envelopes of these very blue remnants occupy exceptionally small mass fractions of the star.

The relative lifetimes spent in the blue and red stages are shown in Figure 6. Here the initial chemical composition is relatively unimportant in comparison with k, but actually it is the mass-loss rate itself that is the fundamental determining factor. This rate may be theoretically predicted for any value of τ_b/τ_r by evaluating $(M_{\text{initial}} - M_{\text{final}})/\tau_r$, where $M_{\text{initial}}/M_\odot = 30$, $M_{\text{final}}/M_\odot \approx 30 - 23X_e - 75Z_e$, $\tau_r = \tau_{\text{He}}/(1 + \tau_b/\tau_r)$, and $\tau_{\text{He}} \approx (0.17 + 0.4X_e + 0.4Z_e) \times 10^6$ years. If it could be proven that all the observed blue supergiants at these luminosities corresponded to our blue remnants, it would be possible to infer the mass-loss rate or, more simply, k from observations of the relative numbers of blue and red supergiants. The needed value of k, under such a naive assumption (plus the assumption that the later stages of evolution have a negligibly short lifetime owing to neutrino emission), is about 10×10^{-11}. In any case, we may be sure that k is quite large.

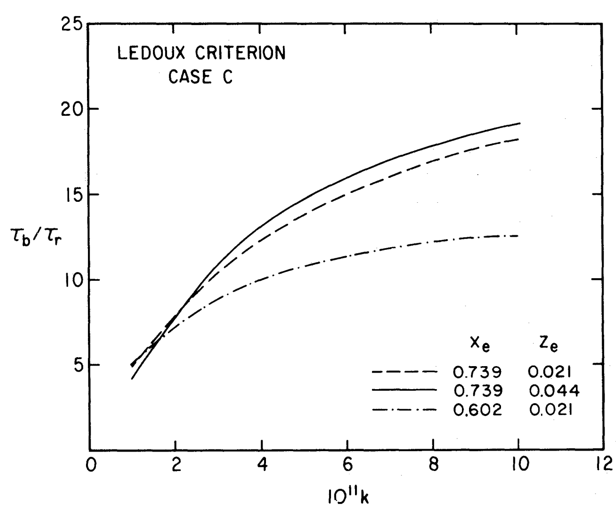

FIG. 6.—Ratio of the lifetimes of the blue and red stages during core helium burning, as a function of initial chemical composition and mass-loss parameter k. The Ledoux criterion for convection has been adopted.

One type of evolutionary track has been omitted from our discussion so far. This type of track occurs for a low value of X_e and Z_e, and resembles the tracks obtained with the use of the Schwarzschild criterion for convection, in that the star remains on the blue side of the H-R diagram during core helium burning, if mass loss is unimportant. If not, only a small amount of mass loss on the main sequence is able to change this blue type of track into a track that resembles the ordinary tracks derived with the Ledoux criterion (see the results for case B evolution in Table 2).

VI. CONCLUSIONS

Stars with masses of 30 $M_\odot$ on the main sequence may in some circumstances lose a considerable fraction of their masses after they evolve into helium-burning supergiants. The present investigation of the evolution of such stars, coupled with our earlier studies, has led to the following conclusions, which, however, are based exclusively on stellar models built with the Cox-Stewart opacities:

1) Stellar luminosities and lifetimes during the phase of core helium burning are affected relatively little by the loss of mass. We have found that $\log (L/L_\odot) = 5.5 \pm 0.2$ and $\tau_{\text{He}}/\tau_{\text{H}} = 0.10 \pm 0.02$ over the whole range of adopted values for the various free parameters.

2) Stellar effective temperatures, on the other hand, are much more sensitive to the loss of mass and to the other free parameters. If the Schwarzschild criterion for convection is adopted, the amount of mass lost during core helium burning is probably small, and the star remains, almost to the end, on the blue side of the H-R diagram, occupying a strip that is separated from the main-sequence band by at least $\delta \log T_e = 0.10$. (Subsequently, the star will evolve into a red supergiant, but its lifetime in that state will be greatly

shortened by neutrino emission and probably also by heavy mass loss.) The same result is obtained with the use of the Ledoux criterion for convection if the initial hydrogen or initial metals abundance is relatively low, specifically, if $X_e + 10Z_e < 0.8$. Otherwise, the star initiates core helium burning as a red supergiant. Rapid mass loss must then be assumed to take place in order to explain the observed paucity of luminous red supergiants. For all of our adopted values of the various free parameters, the star ceases to be a red supergiant after most of the hydrogen envelope has been removed. The blue remnant that is left behind occupies a strip on the H-R diagram which, for reasonably high values of X_e, Z_e, and k, overlaps, and may even extend to the left of, the main-sequence band.

What we have essentially done in the present study is to illustrate, more quantitatively than before, the multiplicity of types of evolutionary tracks that can be obtained with observationally plausible values of the selected free parameters. In our opinion, it is not yet practical to narrow down the choices very much by applying more detailed observational arguments, although we have pointed out a few obvious agreements and disagreements with basic observational data. One major stumbling block is that critically important theoretical parameters, such as stellar opacity and convection, need to be known with greater accuracy.

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